SMC Department of Mathematics
Problem of the Week

Multiples of 11: You are given a special set of numbers that contains all three-digit positive integers that are palindromes. What is the probability that any thee-digit palindrome is also a multiple of eleven?

Solution: A three digit palindrome has the form aba with $a = 1, 2, ..., 9$ and $b = 0, 1, ..., 9$. Thus there are a total of 90 three digit palindromes. We will apply a divisibility test to determine when these numbers are divisible by 11.

A number $x_1x_2x_3....x_n$ is divisible by 11 if the difference between the sum of the digits with odd indices and the sum of the digits with even indices is divisible by 11. Thus $2a - b = 11n$ for some number n. This gives $2a = 11n + b$. The digit a is nonzero, so $1 \leq a \leq 18$. Thus $2 \leq 11n+b \leq 18$ and $n = 0, 1$. If $n = 0$, $2a = b$ and the palindromes are 121, 242, 363, and 484. If $n = 1$, $2a = 11 + b$, then the palindromes are 616, 727, 858, and 979. This give a total of 8 palindromes divisible by 11.

Thus the probability that a three digit palindrome is divisible by 11 is $\frac{8}{90} = 8.\bar{8}\%$.