

## SMC Department of Mathematics

### Problem of the Week

**A Problem in Placing Letters:** Place 4 a's, 4 b's, 4 c's, and 4 d's in a 4-by-4 grid so that there are no identical letters in any row, column, or main diagonal. How many solutions are there?

**Solution:** Each grid is completely determined by the entries in the four corners and along the forward main diagonal. To see this, note that once you decide on the corner entries and the diagonal, the other diagonal is determined. The remaining entries will be along the outer square. Each of those lie in a row or column with two corners and adjacent to a diagonal entry. These three numbers determine the entry.

Filling in the corners clockwise, there are four choices for the upper left corner, three for the upper right, 2 for the lower right, and 1 for the lower left. To complete the forward main diagonal, you need to fill in the two inner entries. This gives two more choices. Multiplying gives  $4 * 3 * 2 * 1 * 2 = 48$  different possible grids.