

SMC Department of Mathematics

Problem of the Week

How far does the ball travel? If an elastic ball is dropped from the Leaning Tower of Pisa at a height of 179 feet from the ground, and on each rebound the ball rises exactly one tenth of its previous height, what distance will it travel before it comes to rest?

Solution: The ball travels 179 feet to the first bounce. Between the first and second bounce, it travels $2(179)(\frac{1}{10})$. The ball continues to bounce, each time reaching $\frac{1}{10}$ of the height of the previous bounce. Thus between bounce n and bounce $n + 1$, the ball travels a distance of $2(179)(\frac{1}{10^n})$. The total distance D is the sum of the distances between each bounce.

$$\begin{aligned} D &= 179 + 2(179)\left(\frac{1}{10}\right) + 2(179)\left(\frac{1}{10^2}\right) + 2(179)\left(\frac{1}{10^3}\right) + \dots \\ &= 179 + 358\frac{1}{10} + 358\frac{1}{10^2} + 358\frac{1}{10^3} \dots \\ &= 179 + 358\left(\sum_{n=1}^{\infty} \frac{1}{10^n}\right) \\ &= 179 + 358\left(\frac{1}{1 - \frac{1}{10}}\right) = 576.\bar{7}ft \end{aligned}$$