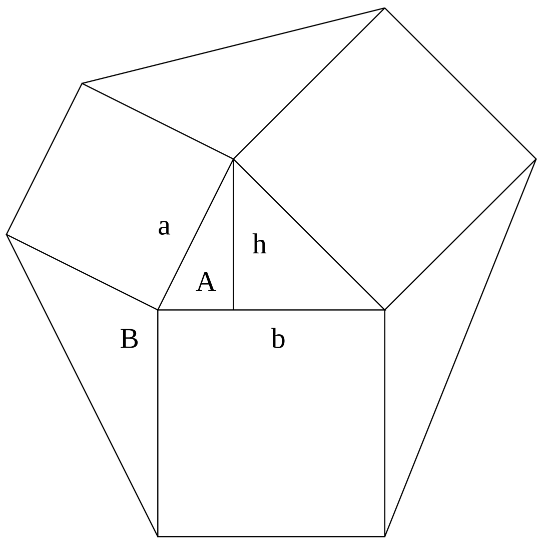
SMC Department of Mathematics
Problem of the Week

Dividing the Hexagon: This weeks problem is from the Pomona College Math Talent Search. The figure below is a hexagon subdivided into four triangles and three squares. Show that each of the triangles has the same area.



Solution: It suffices to show that the area of each of the outer triangles is equal to the area of the inner triangle. I will show the area of the bottom left triangle is equal to the area of the middle triangle. The same argument can be applied to each of the outer triangles. Let a, b be the side lengths of the two squares indicated above, A and B the angles of the triangles separating them (see figure).

The area of a triangle is $\frac{1}{2}(base)(height)$. For middle triangle, take the base to be the side b and the height to be the altitude h. Then $\sin(A) = \frac{h}{a}$, so $h = a \sin A$. Thus the area of the inner triangle is $\frac{1}{2} ba \sin A$. Applying the same reasoning to the outer triangle gives an area of $\frac{1}{2} ab \sin B$. This reduces the problem to showing that $\sin(A) = \sin(B)$.

The two triangles share a vertex with the corners of two squares. Thus $A+B+\frac{\pi}{2}+\frac{\pi}{2} = 2\pi$. Simplifying, we see $B = \pi - A$. Thus $\sin(B) = \sin(\pi - A) = \sin(A)$.