Math 28: Calculus II
Practice Exam 2 Solutions

Justify your answers using relevant terms and results from the course.

1. Compute the given integral using the indicated method.
   (a) $\frac{dx}{\sqrt{6x-x^2}}$ by completing the square.
   Answer: $-\sin^{-1}(1-x/3)+C$.
   (b) $\int \frac{2x^2-5}{x+3}$ by reducing the improper fraction.
   Answer: $-6x+x^2+13\ln(3+x)$.

2. (a) Use tabular integration to compute $\int x^4 \sin x dx$.
   Answer: $(-24+12x^2-x^4)\cos x+(-24x+4x^3)\sin x$.
   (b) Use integration by parts to compute $\int_0^{\frac{1}{\sqrt{2}}} 2x\sin^{-1}(x^2)dx$.
   Answer: $\frac{\pi}{12}+(\frac{\sqrt{3}}{2}-1)$.

3. Use partial fractions to compute $\int \frac{3x-2}{x^2-x-6}dx$.
   Answer: $\frac{7}{5}\ln(x-3)+\frac{8}{5}\ln(x+2)$.

4. Use l'Hôpital's rule to evaluate the given limits.
   (a) $\lim_{x\to 0^+} \frac{\ln x}{x}$.
   Answer: Actually, don't use l'Hôpital. Substitution gives $\frac{-\infty}{0}$, which is a determinate form that implies that the limit is $-\infty$.
   (b) $\lim_{x\to 0^+} \frac{1}{x}-\frac{1}{1-\cos(x)}$.
   Answer: $+\infty$.

5. (a) Evaluate the improper integral $\int_2^\infty \frac{3}{e^x}dx$.
   Answer: $3e^{-2}$
   (b) Determine if the improper integral $\int_2^\infty \frac{3}{2e^x-4}dx$ converges or diverges by combining part (a) and the limit comparison test.
   Answer: Converges.
   (c) Determine if the improper integral $\int_1^\infty \frac{\cos^3(x)}{x^6}dx$ converges or diverges by combining the direct comparison test with the fact that $|\cos(x)|\le 1$ for all real numbers x.
   Answer: Converges.