

Math 27: Calculus I
Extra Credit Review 3
Due Wed Nov 14 for 10 points

Justify your answer using relevant terms and results from the course.

1. **3.5 Trig'**: Compute the derivative of $y = x^2 \sin^4 x + x \cos^{-2} x$.
2. **3.6 Inverse'**: Suppose $f(x) = \sqrt{x} + \sqrt{3x - 3}$.
 - (a) Show that $f^{-1}(5) = 4$ by computing $f(4)$.
 - (b) Find the numerical value of the derivative $f^{-1}'(5)$.
3. **3.7 Implicit Differentiation**: Find the equation of the tangent line at the point $(1, 2)$ to the curve defined by the equation $x^3 + xy + y^2 = 7$.
4. **3.9 Differential Approximation**: Use differentials to estimate the numerical value of $\sqrt[3]{27.2}$.
5. **4.1 Graph sketching**:
 - (a) Sketch the graph of $y = 3x^4 - 4x^3 + 30$. Indicate where the graph is increasing, decreasing, concave up, concave down, as well as the precise location of all critical and inflection points.
 - (b) Based on your graph in part (a), what are the global maximum and minimum values?
6. **4.3 Extreme values**: Determine the global extreme values of the function $f(x) = -2x^3 + 9x^2 - 12x + 4$ on the interval $[0, 2]$.
7. **4.4 Marginality**: Suppose the revenue from selling q items is $R(q) = 250q - \frac{1}{4}q^2$. Suppose the cost of producing q items is $C(q) = 150 + 5q$.
 - (a) Use differentiation to produce formulas estimating the marginal revenue MR and the marginal cost MC .
 - (b) Find the quantity of items to produce that will maximize profits.
8. **4.5 Optimization**: A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

9. **4.6 Related rates:** A 5 foot ladder is leaning against a house when its base starts to slide away. When the base is 4 feet from the house, the base is moving away at a rate of 2 ft/sec.

- (a) How fast is the top of the ladder sliding down the wall at that moment?
- (b) At what rate is the angle θ between the ladder and the ground changing then?

10. **5.1 Computing distance:** A care comes to a stop 6 seconds after the driver applies the brakes. While the brakes are on, the following speeds are recorded:

Time since brakes applied (sec)	0	2	4	6
Speed (ft/sec)	88	40	15	0

- (a) Give a **lower** estimate of distance travelled by the car after the brakes were applied.
- (b) Give an **upper** estimate of distance travelled by the car after the brakes were applied.