

Math 27: Calculus I
Extra Credit Review 1
Due Wed Sep 26 for 10 points

Justify your answer using relevant terms and results from the course. Since there are no calculators allowed on the test, you may write your answers as a numerical formulas where necessary.

1. Exponentials:

- (a) Sketch the graph of an exponential function with initial value 3 and doubling time 8.
- (b) Strontium-90 is a radioactive element with a half-life of 28 years. Suppose 100 mg of Strontium-90 is present in a sample at a given time. How much Strontium-90 will remain 37 years later?

2. Inverse Functions: Consider the function $f(x)$ with the graph given below.

- (a) Determine the numerical values of $f(0)$, $f(1)$, $f(2)$, $f(3)$, and $f(4)$.
- (b) Determine the numerical values of $f^{-1}(0)$, $f^{-1}(1)$, and $f^{-1}(4)$.
- (c) Sketch the graph of $f^{-1}(x)$.

3. Logarithms:

- (a) Given that $\log_2 7 = 2.81$, determine $\log_2 49$ without using a calculator.
- (b) Solve the following equation for x : $8 \cdot 10^x = 2 \cdot e^x$.
- (c) Strontium-90 is a radioactive element with a half-life of 28 years. How long does it take for a sample of Strontium-90 to decay to $1/3$ of its initial quantity? In other words, what is the “third-life” of Strontium-90?

4. Trigonometry:

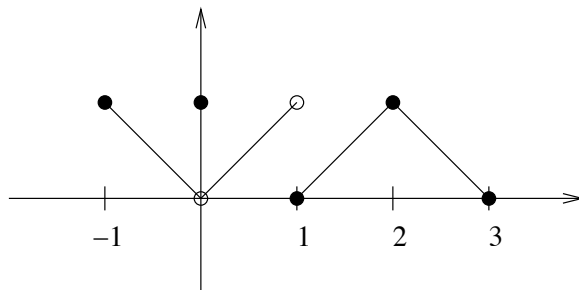
- (a) Tennis star Maria Sharapova is exactly 6 feet tall. Suppose she is standing exactly 10 feet in front of a wall. How far does the flashlight need to be from Maria in the diagram below to make her shadow exactly 24 feet tall?

- (b) In the diagram below, convert the given angles to radians and then find the (x, y) -coordinates of points A, B and C.

- (c) Solve the equation $3 = 4 \tan(2\theta + \pi/6)$ for θ .

5. **Continuity:**

- (a) Show that the equation $x^3 - 4x + 1 = 0$ has a solution for some x -value between 0 and 1.
- (b) Consider the function f defined by the graph below.



- i. Does f have a removable discontinuity at $x = 0$? Explain.
- ii. Does f have a removable discontinuity at $x = 1$? Explain.
- iii. Does f have a removable discontinuity at $x = 2$? Explain.

6. **Limits:**

- (a) Draw one graph of a function f having all the following properties:
- $\lim_{x \rightarrow -\infty} f(x) = -1$,
 - $\lim_{x \rightarrow -1} f(x) = \infty$,
 - $f(0) = 1$,
 - $\lim_{x \rightarrow 1} f(x) = -\infty$,
 - $\lim_{x \rightarrow \infty} f(x) = 0$.
- (b) Suppose that f is a function where $\lim_{x \rightarrow 1} f(x) = 3$. Use the limit laws to show that $\lim_{x \rightarrow 1} (x + 1)f(x) = 6$.
- (c) Evaluate $\lim_{x \rightarrow \infty} \frac{x^2 + 2x - 3}{5x^2 - 4x + 7}$. Justify your steps.
- (d) Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$. Justify your steps.