Math 27: Calculus I
Practice Midterm 1
Worth 10 points extra credit
Due Wed Sept 20 at start of class

Justify your answer using relevant terms and results from the course.

1. Suppose $f(x)$ and $g(x)$ are functions such that $\lim_{x\to 2} f(x) = 7$ and $\lim_{x\to 2} g(x) = 3$. Use the limit rules to show that

$$\lim_{x\to 2} g(x)(x + f(x)) = 27.$$

2. Let $f(x) = \frac{x^2-4}{x^2-5x+6}$. Find the vertical and horizontal asymptotes of the graph $y = f(x)$. Justify your asymptotes by showing the appropriate limit computations.

3. Sketch the graph of a function $f(x)$ satisfying the following conditions:

   (a) $\lim_{x\to-\infty} f(x) = 0$

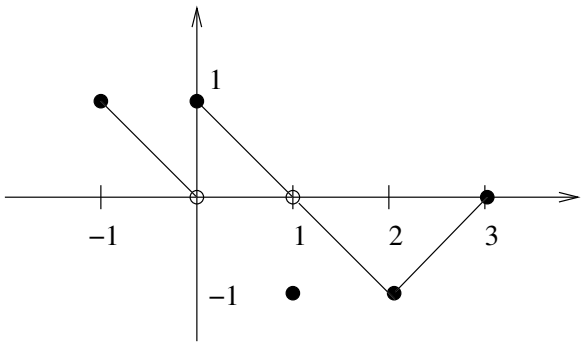
   (b) $\lim_{x\to-2^-} f(x) = -\infty$

   (c) $\lim_{x\to-2^+} f(x) = +\infty$

   (d) $\lim_{x\to 3} f(x) = +\infty$

   (e) $\lim_{x\to+\infty} f(x) = -1$

4. Consider the function f defined by the graph below.



   (a) What is $\lim_{x\to 0} f(x)$?

   (b) Does f have a removable discontinuity at $x = 0$? Explain.

   (c) What is $\lim_{x\to 1} f(x)$?

   (d) Does f have a removable discontinuity at $x = 1$? Explain.

   (e) Is f continuous at $x = 2$? Explain.

5. Suppose that the position of a ball at time t (in seconds) is given by $y = \sqrt{2t+1}$.

   (a) What is the average speed of the ball over the time interval $[3, 12]$?

   (b) What is the instantaneous speed of the ball at time $t = 3$?

6. Show that the derivative of $f(x) = x^2 - 3x + 5$ is $f'(x) = 2x - 3$ using the definition of the derivative.