Math 27: Calculus I
Workshop 6: Integration
Due Monday, December 6

The solutions write-up is equal in course credit to one homework assignment. Each group should turn in one write-up, due at the start of the final exam on Monday, December 6. Justify all answers using relevant terms and results from the course.

1. Label each of the following as TRUE or FALSE. Explain your reasoning.
   (a) $\int_a^b [f(x)g(x)]\,dx = (\int_b^a f(x)\,dx)(\int_a^b g(x)\,dx)$.
   (b) $\int_{-5}^{5}(ax^2+bx+c)\,dx = 2\int_0^5(ax^2+c)\,dx$.
   (c) $\int_{-2}^{1}\frac{1}{x^4}\,dx = -\frac{3}{8}$.
   (d) $\int_0^2(x-x^3)\,dx$ represents the area under the curve $y = x - x^3$ from 0 to 2.
2. Evaluate the following integrals.
   (a) $\int_{-1}^{2} x^2 - 3x + 3\ dx$.
   (b) $\int_1^4 \sqrt{x}\ dx$.
   (c) $\int_0^{\sqrt[3]{\pi}} x^2\sin(x^3)\,dx$.
   (d) $\int_2^7 \frac{x-1}{\sqrt{x+2}}\,dx$.
3. Evaluate $\int_1^4 \frac{2}{3}(x-1)\,dx$ in each of the following three ways.
   (a) By sketching the graph of the integrand and perform an area computation based on your sketch.
   (b) By using the fundamental theorem of calculus.
   (c) By directly evaluating the defining limit of Riemann sums.

4. Let $f(x) = 4x - x^2$.

   (a) Draw a graph of this function on the interval [0,4].

   (b) Partition the interval [0,4] into eight sub-intervals. Estimate the function f by another function g which is constant on each of these sub-intervals; that is pick an actual constant value for each sub-interval which you think best represents f on that sub-interval.

   (c) Find the average value of g on the interval [0,4].(Hint: On any sub-interval, g is constant, so there its average value is a constant. Notice what fraction of the whole interval each sub-interval takes up. How will you use this information in your computation?)

   (d) We define the *average value of a function f on an interval* $[a, b]$ to be

   $$f_{avg} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

   Use this definition to compute the average value of the function $f(x) = 4x - x^2$ on the interval $[0, 4]$. How close was your estimate from part (c)?

5. Let f be defined on $0 \leq x \leq 1$ by the following rule: $f(x)$ is the first digit in the decimal expansion for x. For example, $f(\frac{1}{2}) = 5$ and $f(0.719) = 7$.

   (a) Sketch the graph of $y = f(x)$. Be careful to use an appropriate scale for x and y.

   (b) Compute the definite integral

   $$\int_0^1 f(x)\, dx$$

   (c) Now define the function g by declaring $g(x)$ to be the second digit in the decimal expansion for x. What is $g(0.293)$?

   (d) Compute $\int_0^1 g(x)\, dx$

6. What is wrong with the following argument?

$$\int_{-1}^{2} \frac{dt}{t^2} = \frac{-1}{t}\bigg|_{-1}^{2} = \frac{-1}{2} - (\frac{-1}{-1}) = \frac{-1}{2} - 1 = \frac{-3}{2}$$