Math 185
Assignment 10
Due Wednesday Nov 15

1. Evaluate the following integrals using the Cauchy integral formula:

   (a) $\int_C \frac{z^2}{z-1}\,dz$ where C is a circle of radius 2, centered at 0.

   (b) $\int_C \frac{e^z}{z^2}\,dz$ where C is the unit circle.

   (c) $\int_C \frac{dz}{z^3}$ where C is the square with vertices $-1-i$, $1-i$, $1+i$, $-1+i$.

   (d) $\int_C \frac{\sin e^z}{z}\,dz$ where C is the unit circle.

   (e) $\int_C \frac{\sin z}{z^4}\,dz$ where C is the unit circle.

2. Suppose that $f(z)$ is analytic except possibly at 2 points z_0 and z_1. Let C be a big simple closed curve encompassing z_0 and z_1, and let C_0 and C_1 be small circles encompassing only z_0 and z_1, respectively. Use the deformation theorem to show that

$$\int_C f(z)\,dz = \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz.$$

3. Use # 2 above and the Cauchy integral formula to evaluate the integrals:

   (a) $\int_C \frac{z^2-1}{z^2+1}\,dz$ where C is a circle of radius 2 centered at 0.

   (b) $\int_C \frac{dz}{z^2-2i}$ where C is the circle of radius 2 centered at 1.

   (c) $\int_C \frac{dz}{z^2(z^2+16)}$ where C is the circle of radius 2 centered at 0.

4. Let $f : A \to \mathbb{C}$ be analytic, A open, and let C be a simple closed curve in A. For any $z_0 \in A$ not on A, show that

$$\int_C \frac{f'(\xi)}{\xi - z_0}\,d\xi = \int_C \frac{f(\xi)}{(\xi - z_0)^2}\,d\xi.$$

5. Needham page 446 # 1, 3, 4, 5.