

Math 10 Extra Credit Review 2  
10 points due Wed Apr 25 in class

1. Suppose that when firms X and Y sell product at prices  $p_x$  and  $p_y$ , respectively, their yearly sales  $q_x$  and  $q_y$  are given by the formulas

$$\begin{aligned}q_x &= 44 - 2p_x + p_y, \\q_y &= 44 - 2p_y + p_x.\end{aligned}$$

If firm X has a production cost of \$6/unit, and firm Y has a production cost of \$6/unit, then the yearly profits are given by the formulas

$$\begin{aligned}b_x &= (p_x - 6)q_x, \\b_y &= (p_y - 6)q_y.\end{aligned}$$

Find a Nash equilibrium of prices.

2. Allegra and Basil are to fight a three stage duel with toy suction-cup guns. In stage 1, each gun is loaded with a single suction-cup, and they are positioned 20 feet apart. Either may fire his or her gun. In stage 2, they advance to be 10 feet apart. Either may fire his or her gun, if they have a suction-cup remaining. In stage 3, they advance to be 0 feet apart. Either may fire his or her gun, if they have a suction-cup remaining. The duel ends either when at least one player is hit or if neither fires at the end of the 0 ft stage.

When they are 20 feet apart, each has a  $3/7$  chance of successfully hitting the other. When they are 10 feet apart, each has a  $5/7$  chance of successfully hitting the other. When they are 0 feet apart, each has a 100% chance of successfully hitting the other.

Each player gets a payoff of -1 if he/she is hit while the other is not hit. Each player gets a payoff of +1 if he/she is not hit, while the other is hit. Each player gets a payoff of 0 if both are hit or neither is hit.

Find a Nash equilibrium of pure-strategies for the duel specifying in each stage whether the player waits or shoots.

3. Determine maximin mixed-strategies producing a Nash equilibrium for the tennis-shot game given by the table below.

Success %	DL	CC
DL	(40,60)	(80,20)
CC	(70,30)	(20,80)

4. Use best-reply analysis to find all Nash equilibria of mixed-strategies for the battle-of-the-buddies game given by the table below.

Satisfaction	Starbucks	Peet's
Starbucks	(4,1)	(0,0)
Peet's	(0,0)	(1,4)

5. What is the strategic effect of allowing the row player to send a single text message to the column player prior to game play in the *Battle of the Buddies* given in problem 4?
6. Suppose actors come in two types: good and bad. Both types of actors can get media hype if the studios promote them by using a public relations (PR) firm, but media hype is more costly to obtain for bad actors since they require more promotion. Suppose that studios have to spend  $C$  dollars on PR to get media hype for good actors, and  $20C$  dollars on PR to get media hype for bad actors. Actors with media hype profit the studios \$50,000,000. Actors without media hype profit the studios \$6,000,000. What is the range of  $C$  values for which studios will choose to generate media hype for good actors, but not for bad actors?