

Math 10: The Art and Practice of Mathematics
Extra Credit (20 points) Practice Final
Due at end of class on Fri May 12
Solution will be released at end of class on Wed May 10

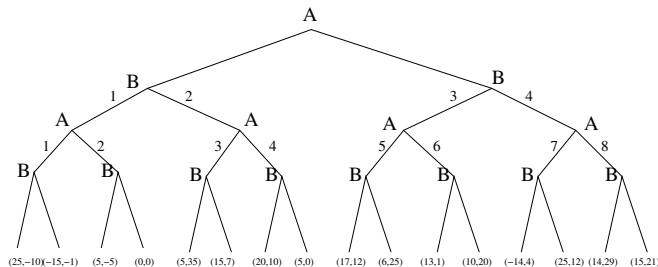
Justify your answers using relevant terms and methods from the course.

1. (*It is not necessary to submit a solution to this one to get full extra credit.*)
A random question of the type found on midterm 2. Practice midterm 2 and its solution may be found at

<http://galileo.stmarys-ca.edu/bdavis/math10/math10.html>

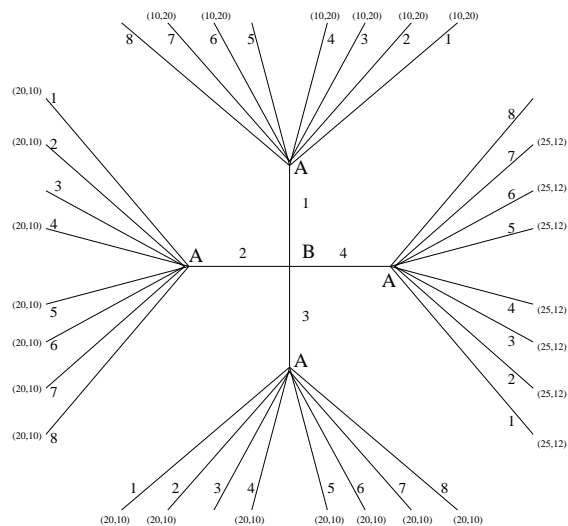
2. Suppose motels come in two types: clean and smelly. Both types of motels can get annual AAA certification, but for the smelly motels certification is more costly to obtain. Suppose that clean motels have to spend C dollars annually preparing for certification; smelly motels have to spend $3C$ dollars annually preparing for certification. AAA certified motels can earn \$2,000,000 annually. Motels without AAA certification earn only \$1,250,000 annually. What is the range of C values for which a clean motel will chose to signal with this device but a smelly motel will not?
3. Suppose that IBM has an IT problem which, if solved, will increase IBM's profits by \$2,400,000. IBM wishes to outsource solving the IT problem to Google. Google charges \$400,000 for standard service, for which Google makes a profit of \$40,000. Google charges \$600,000 for intensive service, for which Google makes a profit of \$80,000. Notice that if Google charges for intensive service, but provides only standard service, then Google makes a profit of \$240,000. In a project proposal to IBM, Google estimates that standard service has a 60% chance of solving the IT problem, and intensive service has an 80% chance of solving the IT problem.
 - (a) Show that a principal-agent problem arises if IBM hires Google and pays a flat fee up-front for service.
 - (b) Consider the bonus-incentive contract wherein Google provides up-front \$200,000 of noncompensated work to IBM and earns a \$1,000,000 bonus if and only if the IT problem is solved. Does this bonus-incentive contract resolve the principal-agent problem in part (a)?

4. Consider the game tree below.



Suppose B can make a single advance commitment regarding their first move followed by A making a single advance commitment regarding their second move.

- (a) Determine the missing payoffs at the terminal nodes of the advance commitment game tree diagrammed below.



- (b) Find a Nash equilibrium of commitments for the advance commitment game above.
(c) Classify the commitments you found in part (b) above as threats or promises.

5. Consider the Stereo Store Cartel Game given by the game table

Profit in \$100s	D	C
D	(72,72)	(90,54)
C	(54,90)	(81,81)

- (a) Demonstrate that the Stereo Store Cartel Game is a Prisoner's Dilemma.
- (b) Suppose that the stores sign a contract promising cooperation (i.e. promising to play row and column C), and that the penalty for deviating (i.e. playing row or column D) is incurring an immediate fine. Determine if the fine resolves the prisoner's dilemma for each of the amounts \$625, \$1,250, and \$1,875.
6. Suppose that Mobilex and Chevtex are two gas stations that can charge either Low or High prices for gas, and that their monthly profits are given by the Gas Price Game table below.

Profit in \$1000s	Low	High
Low	(246,246)	(253,243)
High	(243,253)	(250,250)

Notice that the Gas Price Game is a Prisoner's Dilemma. Suppose that the interest rate is 4% and that Chevtex is on the verge of bankruptcy; in fact, each month the chance that Chevtex will shutter is $\frac{4}{7}$. Assuming that Chevtex follows the Tit-for-Tat price strategy, determine which of the following Mobilex price strategies is the best: (a) Never defect, (b) Defect only once, (c) Defect forever.