

Math 10: The Art and Practice of Mathematics
Assignment 11
Due Wednesday May 10

Your solutions should be written so-as to be clear to an audience of fellow math 10 students.

Recall that to demonstrate that a game has a *prisoner's dilemma*, we proceed using the *Prisoner's Dilemma (PD) Criteria* as follows. First, one must designate a hypothetical play as *total defection* (denoted H_D) and another hypothetical play as *total cooperation* (denoted H_C). Then we check that:

(i) Total defection H_D is a Nash equilibrium. To find a Nash equilibrium one might need to consider mixed strategies.

(ii) Both players prefer the payoff of total cooperation H_C to total defection H_D . In other words, there is *mutual gain through cooperation*.

(iii) Both players have an *incentive to defect*; i.e. both players have an incentive to deviate from the strategies of H_C to the strategies of H_D .

1. For the game tables below, determine if the game has a prisoner's dilemma, and then demonstrate your answer is correct using the PD Criteria describe above.

(a) The Cartel Game:

| Profit in \$100s | D | C |
|------------------|-----------|-----------|
| D | (288,288) | (360,216) |
| C | (216,360) | (324,324) |

(b) The Battle of the Buddies:

| Satisfaction | Starbucks | Peet's |
|--------------|-----------|--------|
| Starbucks | (4,1) | (0,0) |
| Peet's | (0,0) | (1,4) |

(c) The Tennis-Shot Game:

| Success % | DL | CC |
|-----------|---------|---------|
| DL | (50,50) | (80,20) |
| CC | (90,10) | (20,80) |

(d) Chicken:

| Satisfaction | Chicken | Tough |
|--------------|---------|---------|
| Chicken | (0,0) | (-1,1) |
| Tough | (1,-1) | (-2,-2) |

(e) The Client-Consultant Game:

| \$1000's Profit | Routine Effort | Extra Effort if Paid |
|-----------------|----------------|----------------------|
| Routine Offer | (260,10) | (260,10) |
| Extro Offer | (210,60) | (330,20) |

(f) Housemates-Cleaning-the-Kitchen Game:

| Satisfaction | Clean | Don't Clean |
|--------------|--------|-------------|
| Clean | (1,1) | (-1,2) |
| Don't Clean | (2,-1) | (0,0) |

2. Consider the Restaurant Cartel Game given in 1(a) above.

- (a) Suppose that the restaurants sign an iron-clad contract promising cooperation (i.e. promising to play row and column C), and that the penalty for deviating (i.e. playing row or column D) is incurring an immediate fine. Determine if the fine solves the prisoner's dilemma for each of the amounts \$2,500, \$5,000, and \$7,500.
- (b) Suppose we include the fact that litigation is risky and expensive. In particular, suppose that *lawyer's fees* are \$15,000 for each firm and that the loser of a lawsuit pays all lawyer's fees. In the event of a conviction, assume the court will award the plaintiff the *actual damages* of \$10,800 and *punitive damages* of \$10,000. Lastly, suppose that there is a 75% chance of winning a lawsuit over breach-of-contract due to a defection. Does a contract promising cooperation solve the prisoner's dilemma?