Jan 18: Math Goes to Hollywood
Workshop 2
Due Mon Jan 14

Justify your answers using relevant terms and concepts from the course.

1. Using the uniqueness of prime factorization, show that if n^2 is divisible by 5, then n must itself be divisible by 5. Then, using this result, adapt our proof from workshop 1 that $\sqrt{3}$ is irrational to show that $\sqrt{5}$ is irrational.

2. Using the uniqueness of prime factorization, show that if n^2 is divisible by an arbitrary prime p, then n must itself be divisible by p. Then, using this result, generalize your argument in question 1. to show that $\sqrt{p}$ is irrational.

3. Here we speed-up the Euclidean algorithm.

   (a) In today's lecture, we subtracted 184 from the larger number three times in the sequence $(184, 667), (184, 483), (184, 229), (184, 115)$. Explain why we could have done this in one step by finding the remainder of $667 \div 184$ by long division.

   (b) In part (a), we noticed that repeatedly subtracting the smaller number from the larger could be sped up by finding the remainder of dividing the larger by the smaller. Use this remainder technique to explain the following single step of the sped-up Euclidean algorithm: $(459, 85), (85, 34)$.

   (c) Find the GCD of 459 and 85 using the Euclidean algorithm (sped-up if you like) and then reduce the fraction $\frac{85}{459}$ to lowest terms.

   (d) Find the GCD of 73656 and 28224 using the Euclidean algorithm (sped-up if you like) and then reduce the fraction $\frac{28224}{73656}$ to lowest terms.