Jan 18: Math Goes to Hollywood
Workshop 4
Due Mon Jan 22

Justify your answers using relevant terms and concepts from the course.

Recall that in lecture we claimed that

$$\sqrt{3} = [1; 1, 2, 1, 2, 1, 2, \ldots].$$

1. Evaluate the truncated continued fractions $[1; 1]$, $[1; 1, 2]$, $[1; 1, 2, 1]$, $[1; 1, 2, 1, 2]$ to obtain rational approximations of $\sqrt{3}$.

2. Use a calculator to compute the decimal values of the fractions you obtained in question 1. Compare with the decimal value of

$$\sqrt{3} = 1.7320508\ldots$$

   and state if the longer continued fractions give better approximations of $\sqrt{3}$.

3. In this question we speed up the computations of question 1. Suppose that the continued fraction $[1; 1, 2, \ldots, 1, 2]$ evaluates to the fraction a/b. Then (amazingly) the longer continued fraction that has an extra 1 and 2 stuck on the end, $[1; 1, 2, \ldots, 1, 2, 1, 2]$, will evaluate to $\frac{2a+3b}{a+2b}$. Start with the fraction you computed for $[1; 1, 2, 1, 2]$ in question 1, and use the rule to rederive the fraction for $[1; 1, 2, 1, 2, 1, 2]$. Check against the answer you got in question 1.

4. Starting with the fraction for $[1; 1, 2, 1, 2, 1, 2]$, use the rule given in question 3 to evaluate the continued fraction $[1; 1, 2, 1, 2, 1, 2, 1, 2]$ and $[1; 1, 2, 1, 2, 1, 2, 1, 2, 1, 2]$. Use a calculator to compute the decimal value of the last continued fraction and compare with the decimal value of $\sqrt{3}$ given in question 2.

5. In this problem we justify the rule given in question 3. Start by assuming that the continued fraction

$$[1; 1, 2, \ldots, 1, 2] = a/b$$

i.e. the continued fraction evaluates to a regular fraction, which we have written in the generic form a/b.

(a) Show that adding 1 to both sides gives

$$[2; 1, 2, \ldots, 1, 2] = \frac{a+b}{b}.$$

(b) Next explain why taking reciprocals of both sides gives

$$[0; 2, 1, 2, \ldots, 1, 2] = \frac{b}{a+b}.$$

(c) Then show that adding 1 to both sides gives

$$[1; 2, 1, 2, \ldots, 1, 2] = \frac{a+2b}{a+b}.$$

(d) Next explain why taking reciprocals of both sides again gives

$$[0; 1, 2, 1, 2 \ldots, 1, 2] = \frac{a+b}{a+2b}.$$

(e) Finally show that adding 1 to both sides gives

$$[1; 1, 2, 1, 2, \ldots, 1, 2] = \frac{2a+3b}{a+2b}.$$