Math 112
Assignment 7
Due Wed Apr 16

1. Suppose $L : K$ is a field extension and let $G = Gal(L : K)$ be the Galois group.

   (a) Let $H \subset G$ be a subgroup. Define the *fixed points* of H to be

   $$H^\dagger = \{\alpha \in L | \phi(\alpha) = \alpha \text{ for all } \phi \in G\}$$

   Show that $H^\dagger$ is a subfield of L.

   (b) Given subgroups H_1 and H_2 of G such that $H_1 \subset H_2$, prove that

   $$H_1^\dagger \supset H_2^\dagger$$

   (c) Given an intermediate field M of $L : K$, recall that, by definition $M^* = Gal(L : M)$. Explain why M^* is a subgroup of G.

   (d) Given intermediate field M_1 and M_2 of $L : K$ such that $M_1 \subset M_2$, prove that

   $$M_1^* \supset M_2^*$$

2. Stewart 8.1-8.3, 8.12(a-d)