

Math 10: The Art and Practice of Mathematics

Probability Workshop

Here we will perform some simple experiments involving flipping coins and rolling dice. In each case, we will collect data and then analyze the data to determine the frequency of certain outcomes. For example, suppose we flip a coin 7 times and observe the following outcomes:

HTTHHTH

Notice that we obtained 4 heads and 3 tails. The *frequency* of heads is $4/7$. Using a calculator we find that $4/7 = .5714\dots$ and so we may express the frequency of heads as a percentage by 57%. The frequency of tails in this experiment is $3/7$ or 43%.

1. **Biased coins?** A flipped coin has an equal chance of giving heads and tail, right? So if we repeatedly flip four different coins we expect each coin to give a different sequence of flips, right? Try the following experiment. Flip four coins simultaneously and write down the outcome for each coin. For example

Trial	Coin 1	Coin 2	Coin 3	Coin 4
1	T	H	H	T
2	H	T	H	T
3	T	T	H	H
\vdots	\vdots	\vdots	\vdots	\vdots

Repeat this experiment for 50 trials. Analyze the data as follows: In a given trial, if 3 or 4 coins all flip the same, mark that as a "coincidence." What was the frequency of "coincidences" in your experiment? Do you think the 4 coins tend towards giving the same sequence of flips?

2. Deal or No Deal?

Suppose six briefcases each contain the six different amounts of money: \$0, \$150,000, \$200,000, \$1,000,000, \$300,000, \$500,000.

You are about to roll a 6-sided dice which will determine which of the briefcases to open for your prize, when you are offered a deal to take \$400,000 to walk away. Should you take this deal?

Perform the experiment 50 times of rolling the dice and record which prize you win. Add of the grand total of all prize money won. What was the *average* amount of money won per game? Would you have been better off taking the deal 50 times or rolling the die 50 times?

3. **A fair game?** Consider the following two player game: Player A and player B each flip a coin. If both coins show heads, player A wins \$3 from B. If both coins show tails, player A wins \$1 from B. If one coin shows a head and the other a tail, player B wins \$2 from player A.

Play this game 50 times, keeping track of the grand total of money won or lost by each player. Based on winnings, do you think this is a *fair game*, or does it appear to favor one of the players?

4. **A rigged game?** Same game as before, except now the player A gets to **choose** whether to show a heads or a tails, instead of flipping the coin. Player B plays as follows: Roll a pair of dice. If you roll an 8, 9, 10 or 11, show heads, otherwise show tails.

Play this game 50 times, keeping track of the grand total of money won or lost by each player. Based on winnings, do you think this is a *fair game*, or does it appear to favor one of the players?