

Math 10: The Art and Practice of Mathematics  
Assignment 5  
Due Fri Mar 28

*Your solutions should be written so-as to be clear to an audience of fellow math 10 students.*

1. For each quadratic equation below, determine the  $(x, y)$ -coordinates of the vertex and make a rough sketch of the graph.

(a)  $y = -(x - 3)^2 - 2$

(b)  $y = -(x - 2)^2 + 5$

(c)  $y = -(1 - x)^2 + 3$

2. For each quadratic equation below, (i) find the critical point using the critical point formula  $x = \frac{-b}{2a}$ , (ii) find the critical value by plugging the  $x$ -value from (i) into the given formula for  $y$ , (iii) make a rough sketch of the graph.

(a)  $y = -x^2 + 6x - 10$

(b)  $y = -x^2 + 20x - 104$

(c)  $y = -x^2 + 7x - (7/2)^2$

(d)  $y = -2x^2 + 12x - 17$

(e)  $y = -2x^2 - 12x - 15$

3. *This problem follows closely the lectures of Wed Mar 12 and Fri Mar 14. It is helpful to consult a copy of the lecture notes, available at <http://galileo.stmarys-ca.edu/bdavis/math10/price.pdf>*

Suppose that firms X and Y sell product at prices  $p_x$  and  $p_y$ , respectively, that their yearly sales  $q_x$  and  $q_y$  are given by the formulas

$$\begin{aligned}q_x &= 50 - 2p_x + p_y, \\q_y &= 50 - 2p_y + p_x.\end{aligned}$$

If firm X has a production cost of \$7/unit, and firm Y has a production cost of \$9/unit, then the yearly profits are given by the formulas

$$\begin{aligned}b_x &= (p_x - 7)q_x, \\b_y &= (p_y - 9)q_y.\end{aligned}$$

- (a) Suppose that firm X charges \$16/unit and firm Y charges \$21/unit. Determine the quantities sold and profits of each firm.
- (b) Suppose that firm Y charges \$21/unit. Determine the best reply price for firm X.
- (c) Determine the best reply formula for firm X.
- (d) Use the formula from part (c) to verify your answer in part (b).
- (e) Determine the best reply formula for firm Y.
- (f) In a single diagram, use your answers in parts (c) and (e) to plot the best reply curves for firms X and Y.
- (g) Use your diagram from part (f) to estimate the Nash equilibrium of prices for firms X and Y.
- (h) Starting with the best reply formulas in parts (c) and (e), use algebra to find the precise values  $p_x$  and  $p_y$  in the Nash equilibrium of prices for firms X and Y. Round your answers to the nearest penny.
- (i) Test your answer from part (h) by verifying that firm X has no incentive to raise or lower prices by 10 cents from the  $p_x$  value in part (h).