Math 10: The Art and Practice of Mathematics
Assignment 2 due Wed Feb 27

Problems 1-4 are related to the Probability Workshop of Fri Feb 15. The corresponding workshop problems are available at the following web address: ***http://galileo.stmarys-ca.edu/bdavis/math10/probability.pdf***

1. **Biased coins?** A *fair* (or *unbiased*) coin has an equal probability of flipping heads or tails. Suppose you flip four coins simultaneously. When three or four coins flip the same, we call that a "coincidence." What is the probability of a coincidence occuring using fair coins?

2. **Deal or No Deal?** Suppose six briefcases each contain the six different amounts of money: \$0, \$150,000, \$200,000, \$1,000,000, \$300,000, \$500,000.

   You are about to roll a 6-sided dice which will determine which of the briefcases to open for your prize, when you are offered a deal to take \$400,000 to walk away. Is the expected payoff of rolling the die greater or less than the \$400,000 deal offered?

3. **A fair game?** Consider the following two player game: Player A and player B each flip a coin. If both coins show heads, player A wins \$3 from B. If both coins show tails, player A wins \$1 from B. If one coin shows a head and the other a tail, player A **loses** \$2 to player B. (This can be represented by player A receiving a payoff of -2.)

   (a) What is the expected payoff of this game for player A?

   (b) Based on your calculation in part (a), is this a fair game, or does it appear to favor one of the players?

4. **A rigged game?** Same game as before, except now the player A gets to **choose** whether to show a heads or a tails, instead of flipping the coin. Player B plays as follows: Roll a pair of dice. If you roll an 8, 9, 10 or 11, show heads, otherwise show tails.

   (a) What is the probability of rolling an 8, 9, 10, or 11 on a pair of dice?

   (b) What is the probability of **not** rolling an 8, 9, 10, or 11 on a pair of dice?

   (c) Suppose player A shows heads. What is the expected payoff of this game for player A?

   (d) Suppose player A shows tails. What is the expected payoff of this game for player A?

   (e) Based on your calculation in parts (c) and (d), is this a fair game, or does it appear to favor one of the players?

5. **Survivor.** The description of the Survivor Immunity game on p.75 gives various numerical probability values which are then assigned to the relevant branches of the game tree on p.74. Let's figure out what happens if Rich increases his estimation of Rudy's strength, meaning that Rich decides that Rudy is harder to beat in the Immunity game than he previously thought. In particular, suppose Rich estimates the probability of an Immunity Challenge win as follows:

   - If Rich chooses to *Give Up*, Kelly has a 70% chance of winning and Rudy has a 30% chance of winning the Immunity Challenge.
   - If Rich chooses to *Continue*, Rich has a 40% chance of winning, Kelly has a 45% chance of winning and Rudy has a 15% chance of winning the Immunity Challenge.

   (a) Relabel the branches of the Survivor Immunity game tree on p.75 with the probability values given above.

   (b) Rollback the Survivor Immunity game tree relabeled in part (a) to determine if Rich should *Give Up* or *Continue*.