Math 10: The Art and Practice of Mathematics
Assignment 10
Due Friday May 9

Your solutions should be written so-as to be clear to an audience of fellow math 10 students.
(I) Recall that to demonstrate that a game has a *prisoner's dilemma*, one must designate a hypothetical play as *total defection* (denoted H_D) and another hypothetical play as *total cooperation* (denoted H_C). Then we check that:

(i) Total defection H_D is a Nash equilibrium. To find a Nash equilibrium one might need to consider mixed strategies.

(ii) Both players prefer the payoff of total cooperation H_C to that of total defection H_D. In other words, there is hypothetically *mutual gain through coordination*.

(iii) Both players have an *incentive to defect*; i.e. both players have an incentive to deviate from the strategies of H_C to the strategies of H_D.

1. For the following game tables below, determine if the game has a prisoner's dilemma, in which case, identify the total defection play H_D and the total cooperation play H_C. If the game doesn't feature a prisoner's dilemma, state which condition above fails: (ii) or (iii).

   (a) The Cartel Game:

Profit in \$100s	D	C
D	(288,288)	(360,216)
C	(216,360)	(324,324)

   (b) The Battle of the Buddies:

Satisfaction	Starbucks	Peet's
Starbucks	(4,1)	(0,0)
Peet's	(0,0)	(1,4)

(c) The Tennis-Shot Game:

Success %	DL	CC
DL	(50,50)	(80,20)
CC	(90,10)	(20,80)

(d) Chicken:

Satisfaction	Chicken	Tough
Chicken	(0,0)	(-1,1)
Tough	(1,-1)	(-2,-2)

(e) The Client-Consultant Game:

$1000's Profit	Routine Effort	Extra Effort if Paid
Routine Offer	(260,10)	(260,10)
Extra Offer	(210,60)	(330,20)

2. Consider the Restaurant Cartel Game given in 1(a) above. Suppose that the restaurants sign a contract promising cooperation (i.e. promising to play row and column C), and that the penalty for deviating (i.e. playing row or column D) is incurring an immediate fine. Determine if the fine resolves the prisoner's dilemma for each of the amounts $2,500, $5,000, and $7,500.

(II) Each of the duels below is played in stages. In stage 1, each gun is loaded with a single suction-cup and the players are positioned far apart. Either player may Shoot in stage 1 or Wait for a later stage.

If either player is hit in stage 1, then the game is over: Each player gets a payoff of -1 if s/he is hit while the other is not; Each player gets a payoff of +1 if s/he is not hit when the other is; Each player get a payoff of 0 if both are hit or neither is hit.

If both players Wait in stage 1, then both players advance to be a little closer together and they play another stage of the duel.

1. Amy and Bart fight a duel (described above) with toy suction-cup guns. Their accuracies at scoring a hit at various distances is recorded in the following table.

Distance	Amy's Hit %	Bart's Hit %
18 ft	3/8	3/8
9 ft	5/8	5/8
0 ft	100%	100%

   (a) Find a Nash equilibrium of pure-strategies for the duel specifying in each stage whether the player Shoots or Waits.

   (b) Based on your analysis in part (a), which player is likely to win the duel?

2. Anton and Babette fight a duel (described above) with toy suction-cup guns. Their accuracies at scoring a hit at various distances is recorded in the following table.

Distance	Anton's Hit %	Babette's Hit %
30 ft	7%	37%
20 ft	38%	58%
10 ft	69%	79%
0 ft	100%	100%

   (a) Find a Nash equilibrium of pure-strategies for the duel specifying in each stage whether the player Shoots or Waits.

   (b) Based on your analysis in part (a), which player is likely to win the duel?

   (c) Suppose Anton cracks under pressure and screams that he is going to shoot at the 20 ft stage. What should Babette do to maximize her chance of winning the duel?