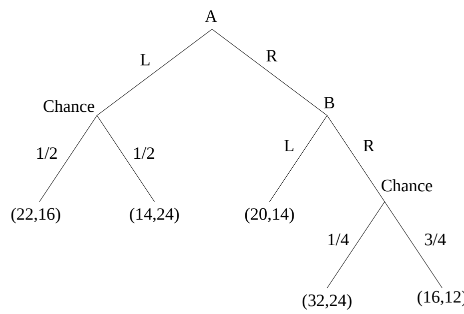
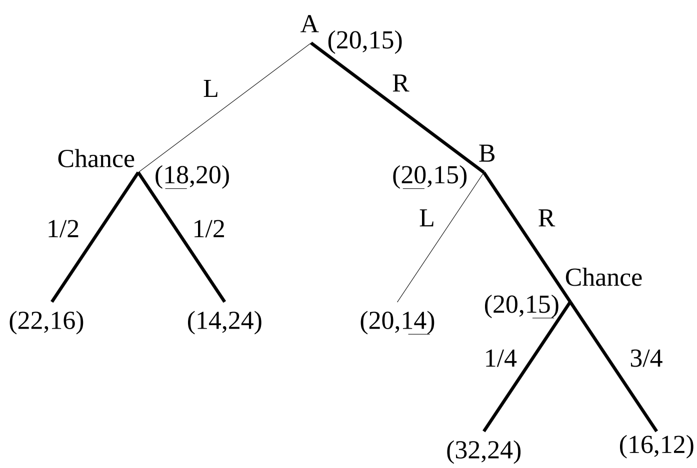
Math 10: The Art and Practice of Mathematics
Worked Examples 1

Justify your answers using relevant terms and results from the course.

1. *Determine the equilibrium strategies and payoffs for each player by rolling back the game tree below.*

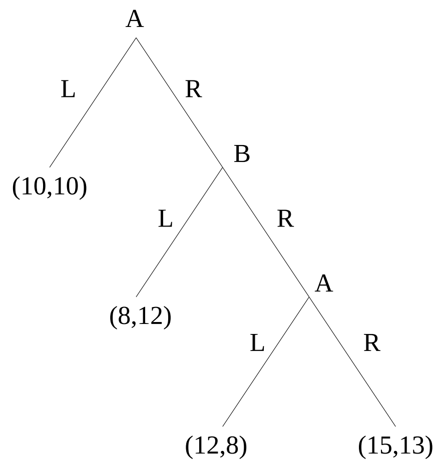


The equilibrium strategies are R for player A, and R for player B. The expected payoff of these strategies is 20 for player A, and 15 for player B. To see this, we rolled-back the game tree as below. In the roll-



back, the Chance node on the L branch of the A node receives payoff $\frac{1}{2}(22, 16) + \frac{1}{2}(14, 24) = (11, 8) + (7, 12) = (18, 20)$, and the Chance node on the R branch of the B node receives payoff $\frac{1}{4}(32, 24) + \frac{3}{4}(16, 12) = (8, 6) + (12, 9) = (20, 15)$.

2. *Construct a game table for the game tree below.*



Taking player A to be the row player, and player B to be the column player, we have the following table of strategies and payoffs:

	L	R
LL	(10,10)	(10,10)
LR	(10,10)	(10,10)
RL	(8,12)	(12,8)
RR	(8,12)	(15,13)

3. *Suppose that Bullseye and Walstore are both planning on opening stores in or near Moraga in 2005. Bullseye has one type of store, which they can locate in town, or just outside of town. Walstore has two types of stores — the regular Walstore and the super Walstore — which, for legal reasons, can only be built just outside of town. A super Walstore attracts more customers than a regular Walstore, but it also costs more to build and operate. The profits made from opening a new Bullseye or Walstore depends on the location and type of the competing store, as expressed in the table below:*

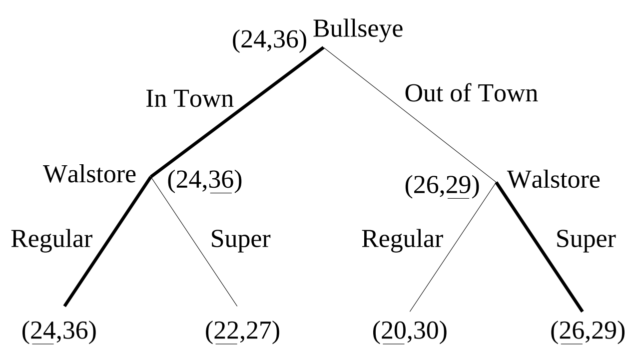
	Bullseye in town	Bullseye outside town
Regular Walstore	(24,36)	(20,30)
Super Walstore	(22,27)	(26,29)

where the numbers represent Walstore and Bullseye's yearly profits in millions of dollars.

For each of the following scenarios, determine the profit maximizing strategy for each chain by drawing and rolling back a game tree.

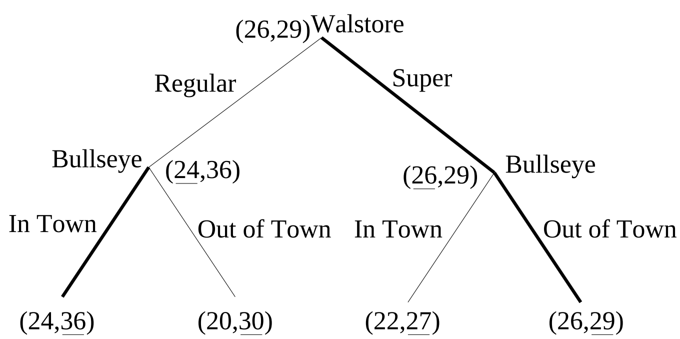
(a) *Suppose the county grants Bullseye a permit to start construction in April 2005, and Walstore a permit to start construction in September 2005.*

The profit maximizing strategies are that Bullseye build in town and Walstore build a regular Walstore. To see this, notice that Bullseye moves (builds) first, and Walstore moves (builds) second. Each chain has a choice of two building options, hence each decision node has two branches. The rolled-back game tree is:



(b) *Suppose the county grants Walstore a permit to start construction in April 2005, and Bullseye a permit to start construction in September 2005.*

The profit maximizing strategies are that Walstore build a super Walstore and Bullseye build out of town. To see this, notice that Walstore moves (builds) first, and Bullseye moves (builds) second. Each chain has a choice of two building options, hence each decision node has two branches. The rolled-back game tree is:



4. *Use best response analysis to find all Nash equilibria in the game table below.*

There are two Nash equiblibria: $H_1 = (\text{row } 3, \text{col } 2)$ and $H_2 = (\text{row } 4, \text{col } 3)$. To see this, in the game table we have boxed the biggest row entry of each column, and the biggest column entry of each row. The Nash equilibria are those cells in which both entries have been boxed, since these are precisely the mutual best replies.

(3,1)	(2,$\boxed{3}$)	(10,0)
(4,$\boxed{5}$)	(3,2)	(6,4)
(2,2)	($\boxed{5}$,$\boxed{4}$)	(9,3)
($\boxed{5}$,6)	(4,5)	($\boxed{12}$,$\boxed{7}$)

5. *Identify the maximin strategy for each player in the game table below. Justify your answer.*

The row maximin strategy is to play row 4, and the column maximin strategy is to play column 2. To see this, we identify the minimum row payoff for each row and the minimum column payoff for each column, as in the table below. We then box the row (or column) which has the maximum payoff among these minimums.

(3,1)	(2,3)	(10,0)	min=2
(4,5)	(3,2)	(6,4)	min=3
(2,2)	(5,4)	(9,3)	min=2
(5,6)	(4,5)	(12,7)	**min=4**
min=1	**min=2**	min=0	

6. *Suppose that when firms X and Y sell product at prices p_x and p_y, respectively, their daily sales q_x and q_y are given by the formulas*

$$\begin{aligned} q_x &= 42 - 2p_x + p_y, \\ q_y &= 46 - 2p_y + p_x. \end{aligned}$$

If firm X has a production cost of \$5/unit, and firm Y has a production cost of \$7/unit, then the daily profits are given by the formulas

$$\begin{aligned} b_x &= (p_x - 5)q_x, \\ b_y &= (p_y - 7)q_y. \end{aligned}$$

Find a Nash equilibrium of prices.

To determine a Nash equilibrium of prices we first must determine best reply price formulas.

Start by substituting the sales formula q_x into the profit formula b_x to obtain

$$b_x = (p_x - 5)((42 + p_y) - 2p_x).$$

Next expand using FOIL and collect p_x powers to obtain

$$b_x = -2p_x^2 + (52 + p_y)p_x - 5p_y - 210.$$

This is a quadratic equation with

$$a = -2,\ b = 52 + p_y,\ c = -5p_y - 210$$

Thus, the critical price point maximizing the b_x-profit is given by

$$\boxed{p_x = \tfrac{-b}{2a} = \tfrac{-(52+p_y)}{2(-2)} = 13 + \tfrac{1}{4}p_y = 13 + .25p_y}$$

We may find the best reply formula for p_y by the same method:

$$\boxed{p_y = 15 + \tfrac{1}{4}p_x = 15 + .25p_x}$$

The graph of a best reply formula is called a *best reply curve*. A Nash equlibrium of prices is given where the best reply curves cross. To determine the coordinates where the best reply curves cross, substitute one best reply formula into the other and solve for the price variable. Substituting the formula for p_y into p_x gives:

$$\begin{aligned} p_x &= 13 + .25p_y \\ &= 13 + .25(15 + .25p_x) \\ &= 13 + 3.75 + .0625p_x \end{aligned}$$

Solve for p_x yields:

$$\begin{aligned} p_x - .0625p_x &= 16.75 \\ .9375p_x &= 16.75 \\ p_x &= \tfrac{16.75}{.9375} \\ p_x &= 17.8666\ldots \end{aligned}$$

To finish, find p_y by substituting in the p_x value:

$$\begin{aligned} p_y &= 15 + .25p_x \\ &= 15 + .25(17.8666\ldots) \\ &= 19.4666\ldots \end{aligned}$$

A Nash equlibrium of prices is $p_x = \$17.87$ and $p_y = \$19.47$.